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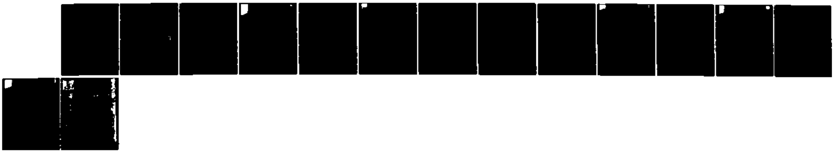

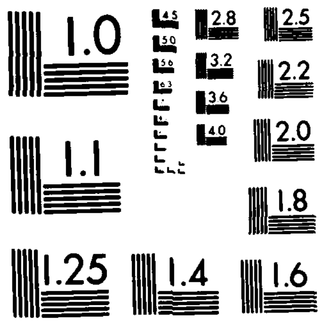

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Statistical Analyses for Nondestructive Testing

D. B. Owen

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Abstract

A statistical method is developed for making an inference about a performance variate based on an observation of a screening variate. This is the typical situation in nondestructive testing where to measure the variable of primary interest would destroy or degrade the item under study. Typically the performance variable is lifetime. The nondestructive testing engineer must look at other related variables and based upon some mathematical analysis and engineering judgments, decide if the item can meet the minimum lifetime requirements.

The method described in this paper allows the engineer to make this inference from the screening variable to the performance variable based on a training set. That is, data are gathered on the screening variable and the performance variable for a set of n items. Then all future items are screened according to the rule developed and among the accepted items there is a preassigned probability that at least a given proportion of the items will have minimum lifetimes. The method is simple and easy to apply.

1 INTRODUCTION

In the typical nondestructive testing situation measurements are made on one random variable while the inference is to a second random variable. For example, a structural part of an aircraft may be X-rayed or measured using ultrasound devices. Based on those measurements we wish to have some high assurance that the aircraft part will last at least for some preassigned length of time.

Until recently the inference has been made based on the presence or absence of any cracks at all showing in the X-ray. However, X-ray technology

has now advanced to the point where cracks are shown which may not have any significant effect on the desired lifetime of the item on test. Packman, et al. (1976) point out the need for statistical methods in handling problems of this type.

A nondestructive testing engineer evaluates the X-ray photographs and based upon his best engineering judgment and what calculations he can make decides whether the structure will last for the required length of time. The method described here allows him to make the inference from the X-rays to the lifetime based on a statistical model. Each item is subjected to a test and some composite measure of the X-ray density is developed which is then correlated with the performance variable (lifetime) which has the specification placed on it. Hence the model takes into account the engineer's judgment through the method of developing the screening variable over a training set; and then the process can be automated. In this way human errors are eliminated from the process. The technique also leads to greater consistency in the judgments that are applied.

Let us be clear at this point that development of this measurement to represent the X-ray will not be trivial. It will take a great deal of cooperative effort between engineers and statisticians to develop meaningful and consistent estimators. Each application will probably require a new development effort.

I do not want to underestimate the effort that will be required to do this properly in each case. On the other hand, in this paper I will assume that the X-measurement (on the correlated variate) and the Y-measurement (on the performance variate), have been developed and are jointly bivariate normally distributed.

In mathematical terms we want to be, say, 99% sure that the remaining lifetime of the part is at least L flying hours.

The technique which will be discussed develops the statistical model for this problem and gives the procedure which must be followed to arrive at the assurance which is sought.

Table 1 gives, for example, the increase in the proportion of aircraft meeting our criterion (in our example, at least L flying hours) if we start off with 70% of the aircraft meeting the criterion and we select a proportion (selection ratio) with the highest scores on their X-rays where the correlation between the X-rays and the lifetime is indicated in the left column. For example, if we choose those aircraft which are in the upper 40% in X-ray scores and we have a correlation 0.75 we will increase our percentage of aircraft with the needed flying time from 70% to 95%. Of course, the use in this context would usually be for special missions. However, the technique which has been developed here is much more general than that.

Now let me show you Table 2 where you can use two criteria to select the aircraft for the special mission. Here the subscript one (1) refers to the performance variable, (lifetime) and the subscripts two and three (2 and 3) refer to the screening variables. This time you may want to think of the second variable as some measure of the structure of the aircraft and the third variable as a measure of the engine viability. The first variable is still the lifetime of the aircraft and we consider that we are successful when the lifetime is at least L flying hours.

Table 1 Proportion meeting requirements after screening δ when proportion meeting requirements before Screening is γ = 0.70

Correlation	Selection Range β						
ρ	0.10	0.20	0.30	0.40	0.50	0.60	0.70
.00	0.70	0.70	0.70	0.70	0.70	0.70	0.70
.10	0.76	0.75	0.74	0.73	0.73	0.72	0.72
.15	0.79	0.77	0.76	0.75	0.74	0.73	0.73
.20	0.81	0.79	0.78	0.77	0.76	0.75	0.74
.25	0.84	0.81	0.80	0.78	0.77	0.76	0.75
.30	0.86	0.84	0.82	0.80	0.78	0.77	0.75
.35	0.89	0.86	0.83	0.82	0.80	0.78	0.76
.40	0.91	0.88	0.85	0.83	0.81	0.79	0.77
.45	0.93	0.90	0.87	0.85	0.83	0.81	0.78
.50	0.94	0.91	0.89	0.87	0.84	0.82	0.80
.60	0.97	0.95	0.92	0.90	0.87	0.85	0.82
.70	0.99	0.97	0.96	0.93	0.91	0.88	0.84
.75	1.00	0.98	0.97	0.95	0.92	0.89	0.86

If the proportion meeting the criterion of at least L flying hours is 0.70 before selection and if the correlation between flying hours and structural strength is $\rho_{12} = 0.5$; correlation between flying hours and engine viability is $\rho_{13} = 0.4$, and the correlation between structural strength and engine viability is $\rho_{23} = 0.1$, then selecting the 40% of all aircraft with 62.1% in the high structural strength and 62.1% in the high engine viability categories will raise the success of the mission from 70% to 88.2%. The technique of having two screening variables is especially important when we have a minimum requirement for two screening variables, as in our example, which cannot be combined into a single variate. In other words for most missions it would *not* be desirable to allow an extra high structural measurement to offset a low engine measurement.

Table 2 Proportion meeting requirements after screening δ when proportion meeting requirements before screening is γ = 0.70

			True Selection Ratio							
			.1	.2	.3	.4	.5	.6	.7	8
			β(.297)	(.430)	(.533)	(.621)	(.685)	(.769)	(.833)	(.892)
ρ_{23}	ρ_{12}	ρ_{13}								
.1	4	.4	943	.912	.886	.861	.841	.813	.768	.763
.1	.5	.4	962	.935	908	.882	.860	.829	.801	.772
.1	5	.5	.978	.954	929	.902	.880	.846	.815	.782
			β(.277)	(.412)	(.518)	(.601)	(.689)	(.761)	(.828)	(.889)
.2	.4	.4	.937	.907	881	859	.833	.810	.787	.762
.2	.5	.4	.957	929	902	.879	.852	.826	.799	.771
2	5	5	.973	.948	923	899	.870	.842	.812	.780
			β(.258)	(.393)	(.502)	(.595)	(.678)	(.754)	(.823)	(.886)
3	.4	4	.932	902	.876	853	.830	.800	.785	.761
3	.5	4	952	.923	.897	.873	848	.823	797	770
3	.5	5	.968	942	917	892	.865	.838	.810	.779

However, there are situations where a combination is quite reasonable. For example, if our concern was only with two structural components on a missile we form a new variable

$$U = \frac{A}{\sigma_{x_2}}(x_2 - \mu_{x_2}) + \frac{B}{\sigma_{x_3}}(x_3 - \mu_{x_3})$$

where (x_2, x_3) are measurements on the two structural components with means (μ_{x_2}, μ_{x_3}) and standard deviations $(\sigma_{x_2}, \sigma_{x_3})$ and where

$$A = \frac{\rho_{12} - \rho_{13}\rho_{23}}{\sqrt{1-\rho_{23}^2}\sqrt{1-\rho_{23}^2-\Delta}}, B = \frac{\rho_{13} - \rho_{12}\rho_{23}}{\sqrt{1-\rho_{23}^2}\sqrt{1-\rho_{23}^2-\Delta}}$$

and

$$\Delta = 1 - \rho_{12}^2 - \rho_{13}^2 - \rho_{23}^2 + 2\rho_{12}\rho_{13}\rho_{23}.$$

Then the correlation between the performance variable (say, Y) and U is given by

$$\rho_{YU} = \frac{\sqrt{1-\rho_{23}^2-\Delta}}{\sqrt{1-\rho_{23}^2}}$$

In other words, it is possible to add measurements of several variables as indicated above and reduce everything to a single screening variables and a single performance variable. This is a viable approach as long as there are not separate minimum requirements for the screening variables. For more information on this see Thomas, Owen and Gunst (1977).

We now will confine the rest of our discussion to a single screening variable (X) and a single performance variable (Y).

2 CASE WHERE ALL PARAMETERS ARE KNOWN

We assume that we have a bivariate normal distribution with a performance variable Y (say, lifetime) and a lower specification limit on Y, which we will designate L. The proportion of the total population of lifetimes (Y) greater than L is designated γ. We propose to screen on the correlated variable X (say, voltage) so that we raise the proportion of Y's greater than L to δ, i.e., in mathematical terms.

$$P\{Y \geqslant L\} = \gamma, \text{ and}$$
$$P\{Y \geqslant L \mid X \geqslant \mu_x - K_\beta\sigma_x\} = \delta,$$

where X and Y have a joint bivariate normal distribution with positive correlation, ρ.

The mean and standard deviation of X are μ_x and σ_x, respectively, and K_β is a standardized normal deviate corresponding to $100\beta\%$ of a standardized

normal distribution in the lower tail of the normal distribution.

Table 3 gives what amounts to the inverse of Table 1. That is, here we set a goal that the proportion δ must meet the requirements on the performance variable and we tabulate the proportion β which must be selected using the screening variable to reach our goal. Table 3 is representative of much more extensive tables which are available in Odeh and Owen (1980).

For example, if we wanted to raise the proportion of acceptable items from 0.75 to 0.95 and the correlation ρ is 0.90 then we would select the upper 68.82% of the X measurements, i.e., select all $X \geqslant \mu_x - 0.4989\sigma_x$.

In our example the original population can be divided into 4 parts:

(1) Those which are accepted by Screening and meet Specifications, i.e., $P\{Y \geqslant L \text{ and } X \geqslant \mu_x - K_\beta\sigma_x\} = \delta\beta = 0.654$.

(2) Those which are rejected by Screening but meet Specifications, i.e., $P\{Y \geqslant L \text{ and } X > \mu_x - K_\beta\sigma_x\} = \gamma - \delta\beta = 0.096$. Note that these two add to $\gamma = P\{Y \geqslant L\}$.

(3) Those which are accepted by Screening but fail Specifications, i.e., $P\{Y \geqslant L \text{ and } X \geqslant \mu_x - K_\beta\sigma_x\} = \beta - \delta\beta = 0.034$.

(4) Those which are rejected by Screening and fail Specifications, i.e., $P\{Y < L \text{ and } X < \mu_x - K_\beta\sigma_x\} = 1 - \gamma - \beta + \delta\beta = 0.216$. Note that these four proportions add to one.

Table 3 Table of values of β where proportion acceptable after selection $= \delta = .950$

Proportion Acceptable in Non-Selected Population $=\gamma$	Correlation $= \rho$						
	.600	.700	.750	.800	.900	.950	1.000
.750	.2812	.4282	.4989	.5661	.6882	.7432	.7895
.760	.3035	.4509	.5206	.5863	.7043	.7567	.8000
.770	.3273	.4745	.5430	.6070	.7205	.7703	.8105
.780	.3527	.4991	.5661	.6281	.7368	.7839	.8211
.790	.3798	.5246	.5898	.6496	.7532	.7975	.8316
.800	.4086	.5511	.6142	.6715	.7696	.8110	.8421
.810	.4392	.5785	.6392	.6938	.7862	.8246	.8526
.820	.4716	.6069	.6648	.7165	.8028	.8381	.8632
.830	.5060	.6362	.6910	.7395	.8194	.8516	.8737
.840	.5423	.6664	.7178	.7628	.8360	.8651	.8842
.850	.5806	.6975	.7450	.7863	.8526	.8784	.8947
.860	.6209	.7293	.7727	.8100	.8691	.8917	.9053
.870	.6629	.7618	.8007	.8338	.8855	.9048	.9158
.880	.7067	.7947	.8288	.8575	.9016	.9177	.9263
.890	.7519	.8280	.8569	.8811	.9176	.9305	.9368
.900	.7981	.8612	.8848	.9043	.9331	.9430	.9474
.910	.8446	.8940	.9121	.9268	.9482	.9552	.9579
.920	.8905	.9256	.9382	.9484	.9627	.9671	.9684
.930	.9341	.9552	.9626	.9684	.9763	.9785	.9789
.940	.9727	.9811	.9840	.9861	.9889	.9894	.9895

This population is then divided into two populations, one of which is accepted by screening:

Those which meet Specifications are

$$P\{Y \geqslant L \text{ given } X \geqslant \mu_x - K_\beta\sigma_x\} = \delta = 0.95.$$

Those which fail Specifications are

$$P\{Y < L \text{ given } X \geqslant \mu_x - K_\beta\sigma_x\} = 1 - \delta = 0.05.$$

And in the population which is rejected by Screening;
Those which meet Specifications are

$$P\{Y \geqslant L \text{ given } X < \mu_x - K_\beta\sigma_x\} = \frac{\gamma - \delta\beta}{1 - \beta} = 0.309.$$

Those which fail Specifications are

$$P\{Y < L \text{ given } X < \mu_x - K_\beta\sigma_x\} = \frac{1 - \gamma - \beta + \delta\beta}{1 - \beta} = 0.691$$

In the original population 25% fail to meet specifications while in the population selected by screening only 5% fail to meet specifications. On the other hand in the rejected group 30.9% do meet specifications.

Now it might be well to digress to remark that we have assumed that we had a lower specification limit and a positive correlation.

The procedure is also applicable for the situations where X and Y are negatively correlated or if there is an upper specification limit, U, on Y. In particular these situations would be handled as follows:

1. Negative correlation and upper specification limit U on Y:
   a. enter the table with the absolute value of correlation,
   b. accept all units whose value X exceeds $\mu - K_\beta\sigma$, and
   c. reject all other submitted units.

2. Negative correlation and lower specification limit L on Y:
   a. enter Table 3 with the absolute value of the correlation,
   b. accept all units whose value X is less than $\mu + K_\beta\sigma$, and
   c. reject all other submitted units.

3. Positive correlation and upper specification limit U on Y:
   a. accept all units whose value X is less than $\mu + K_\beta\sigma$, and
   b. reject all other submitted units.

3. CASE WHERE ALL PARAMETERS ARE UNKNOWN

In most applications the parameters of the distributions will be unknown and they will have to be replaced by estimates obtained from a preliminary sample of size n (a training set). The steps in the screening procedure will now be given. These steps require several tables to be entered to obtain the mathematical quantities required for the procedure. Odeh and Owen (1980) give tables for each of these quantities.

(1) A preliminary sample of size n is obtained of paired values $(x_1, y_1) \ldots (x_n, y_n)$ and the usual estimators of the parameters are computed.

(2) A lower $100\eta\%$ confidence limit on ρ is computed and called ρ^*. If this is positive, we proceed to step (3). If it is negative an upper $100\eta\%$ limit is also computed. If this is positive, the process stops since the two variables X & Y could then be independent. If the $100\eta\%$ upper confidence limit on ρ is also negative, then we proceed, making modifications indicated above in Section 2 for a negative correlation.

(3) A $100\eta\%$ lower confidence limit on $\gamma = P\{Y \geqslant L\}$ is computed and labeled γ^*.

(4) Enter a table of the normal conditioned on T distribution with parameters (and estimates) and degrees of freedom $= n - 1$,

$$\gamma^*,$$

$$\rho^* \sqrt{\frac{n}{n+1}},$$

$$\delta.$$

This table is like Table 3 except that it also varies with degrees of freedom.

(5) All product items are accepted if

$$X \geqslant \bar{x} - t_\beta \sqrt{\frac{n+1}{n}} s_x$$

(6) We can then be at least $100(2\eta - 1)\%$ sure that at least $100\delta\%$ of the Y's are above L in the selected population.

For example, if a preliminary sample of size 17 is taken and $r = 0.94$ then choosing $\eta = .95$ we obtain a 95% lower confidence limit on ρ to be $\rho^* = 0.8558$.

If $k = (\bar{y} - L)/s_y = 2.0$ then a 95% lower confidence limit on γ is $\gamma^* = 0.90$.

We enter the normal conditioned on t-table with (16,0.90, 0.8317,0.95) for $(f, \gamma, \rho, \delta)$ and obtain $t_\beta = 1.384$. Our criterion is to select all items for which $X \geqslant \bar{x} - 1.424 s_x$. Then in the selected group we can be at least 90% sure that at least 95% of the performance variable, Y, is greater than the lower specification limit L.

If this screening is performed on a finite group of say M, items then the items in that group follow a binomial distribution with parameters M and δ. The situation is very similar to what is called prediction intervals in the literature, except that we say we are at least 100 $(2\eta - 1\%)$ sure that the probability of z or less defectives is *at least* that given by the binomial distribution. Hence, if $M = 10$ for the example above with an $\eta = .95$, then we are at least 90% sure that the probability of zero defectives in this group is 0.5999. See the paper by Owen, Li and Chou (1981) for more details on this.

4. TWO-SIDED SPECIFICATION LIMITS, KNOWN PARAMETERS

Now let us consider extensions of these procedures to two-sided limits, i.e., where we are interested in controlling the $P\{L \leqslant Y \leqslant U\}$. Here things become much more complicated and it is convenient to define a γ_1 and γ_2 which are the γ's of the one-sided procedure, i.e., let

$$P\{Y \geqslant L\} = \gamma_1 \qquad L = \mu_y - K_{\gamma_1} \sigma_y$$
$$P\{Y \leqslant U\} = \gamma_2 \qquad U = \mu_y + K_{\gamma_2} \sigma_y.$$

Before screening, the proportion of acceptable product is $\gamma_1 + \gamma_2 - 1$. After screening, $\gamma_1 + \gamma_2 - 1$ is raised to δ.

Because of the many combinations of γ_1 and γ_2 possible we sought a solution first when

$$\gamma_1 = \gamma_2 = \gamma.$$

In this case we accept all items for which

$$\mu_x - K_\beta \sigma_x \leqslant X \leqslant \mu_x + K_\beta \sigma_x$$

where K_β is read from tables obtained from solving

$$BVN(K_\gamma, K_\beta) - BVN(K_\gamma, -K_\beta) - \frac{\delta + 1}{2}(2\beta - 1) = 0,$$

where BVN is the bivariate normal cumulative with the variates standardized.

Table 4 gives values of K_β that solves this equation when $\delta = 0.90$ More extensive tables may be found in Li and Owen (1979).

Note that in the one-sided case we can select so few product that we can raise δ to any preassigned value. That is, there is no mathematical limit on δ; although there are clearly practical ones depending upon how much good pro-

Table 4 Table of K_β for two-sided specifications with $\gamma_1 = \gamma_2 = \gamma$ and proportion acceptable after selection of $\delta = 0.90$

Gamma	Correlation		
	.90	.95	1.00
.78	.3373	.6657	.8820
.79	.4368	.7231	.9239
.80	.5252	.7814	.9674
.81	.6085	.8411	1.0129
.82	6894	.9026	1.0606
.83	.7699	.9664	1.1108
.84	.8517	1.0329	1.1639
.85	.9357	1.1029	1.2206
.86	1.0234	1.1771	1.2816
.87	1.1159	1.2565	1.3476
.88	1.2151	1.3424	1.4202
.89	1.3231	1.4370	1.5011
.90	1.4432	1.5428	1.5932
.91	1.5804	1.6644	1.7013
.92	1.7438	1.8102	1.8339
.93	1.9522	1.9983	2.0099
.94	2.2622	2.2842	2.2865

duct you are willing to screen out. For the two-sided case the largest value that δ can attain is

$$2G\left(\frac{K_\gamma}{\sqrt{1-\rho^2}}\right) - 1.$$

When $\gamma_1 \neq \gamma_2$ we accept all items for which

$$\mu_x - K_{\beta_1}\sigma_x \leqslant X \leqslant \mu_x + K_{\beta_2}\sigma_x$$

where K_{β_1}, K_{β_2} are obtained from tables in equal-tailed cases. The proportion of Y's meeting the specification is

$$\begin{aligned}\Delta &= P\{-K_{\gamma_1} \leqslant Z_1 \leqslant K_{\gamma_2} | -K_{\beta_1} \leqslant Z_2 \leqslant K_{\beta_2}\} \\ &= \delta - \frac{1}{\beta_1 + \beta_2 - 1} \int_{K_{\gamma_1}}^{K_{\gamma_2}} \int_{-K_{\beta_2}}^{-K_{\beta_1}} g(u,v)\, du dv \\ &= \delta - \text{adj. } \delta,\end{aligned}$$

where $g(u, v)$ is the density function for a standardized bivariate normal distribution.

We then computed values of this adjustment to δ and obtained the results given in Table 5.

Table 5 Values of maximum adjustments to δ when using equal tail specifications in unequal tail cases

$\delta = 0.90$	adj. δ is largest when $\rho = 0.70$		
	γ_1	γ_2	adj. δ
	.88	.89	.004309
	.88	.90	.006953
	.88	.91	.008708
	.88	.92	.009912
	.88	.93	.010772
	.88	.94	.011446
	all other combinations of ρ and γ_1, γ_2		
	max. adj. $\delta = 0.006$		
$\delta = 0.95$	max. adj. $\delta = 0.006$		
$\delta = 0.99$	max. adj. $\delta = 0.001$		

Now let us illustrate the procedure by an example:

Y = voltage at an internal point of a device

X = voltage at an external point of a device

$L = 12$ volts $\quad U = 16$ volts

$\mu_y = 13.8$ volts $\quad \sigma_y = 2.13$ volts $\quad \rho = 0.90$

Then,

$$K_{\gamma_1} = 0.845 \qquad K_{\gamma_2} = 1.033$$

or

$$P\{Y \geqslant L\} = \gamma_1 = 0.80, \quad P\{Y \leqslant U\} = \gamma_2 = 0.85$$

For $\delta = 0.90$, we get $K_{\beta_1} = 0.5252$, $K_{\beta_2} = 0.9357$ and we will accept all items for which

$$\mu_x - 0.5252\sigma_x \leqslant X \leqslant \mu_x + 0.9357\sigma_x$$

In the selected group, at least 89.4% of the Y values will be between 12 volts and 16 volts.

5. TWO SIDED SPECIFICATION LIMITS AND UNKNOWN PARAMETERS

When parameters are unknown we start with a training set of size n and proceed through the following four steps:

1. Find γ_1^*, γ_2^* such that

$$P\{\gamma_1 \geqslant \gamma_1^*, \gamma_2 \geqslant \gamma_2^*\} = \eta$$

where γ_1^*, γ_2^* can be obtained from a table of a bivariate noncentral t-distribution. See Owen (1965) for a short table of this and see Owen and Frawley (1971) for a larger table.

2. Find ρ^* such that

$$P\{\rho \geqslant \rho^*\} = \eta.$$

Odeh and Owen (1980) give a table for obtaining this easily.

3. If σ_x is known, enter table with $\left(\rho^* \sqrt{\frac{n}{n+1}}, \delta, \gamma_1^*\right)$ to obtain $K_{1_{\beta_1^*}}$ and similarly, to obtain $K_{2_{\beta_2^*}}$. Compute $K_{\beta_1^*} = \sqrt{\frac{n+1}{n}}\, K_{1_{\beta_1^*}}$, $K_{\beta_2^*} = \sqrt{\frac{n+1}{n}}\, K_{2_{\beta_2^*}}$. If σ_x is unknown, tables have not yet been prepared.

4. Select all X for which

$$\bar{x} - K_{\beta_1^*}\sigma_x \leqslant X \leqslant \bar{x} + K_{\beta_2^*}\sigma_x.$$

Then, in the selected population we will be at least $100(2\eta - 1)\%$ confident that approximately $100\delta\%$ of the Y's are between L and U.

6. CONCLUSION

The screening procedures given here are all based on a bivariate normal model. If such a model does not obtain, the solution seems to lie in the direction of transformations to bivariate normality. A comprehensive approach to such transformations is needed.

A great deal of work will probably be required for each application in devising the variable or variables to be used as screening variables. This also is a topic for further work.

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